SusHi 1.1.0: A program for the calculation of Higgs production in
gluon fusion and bottom-quark annihilation in the Standard Model,
the 2HDM and the MSSM

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Abstract
This article describes the code SusHi (for “Supersymmetric Higgs”) [1] which calculates the
cross sections $pp/p\bar{p} \rightarrow \phi + X$ in gluon fusion and bottom-quark annihilation in the SM,
the 2HDM and the MSSM, where $\phi$ is any of the neutral Higgs bosons within these models.
Apart from inclusive cross sections up to NNLO QCD, differential cross sections with respect
to the Higgs transverse momentum $p_T$ and (pseudo-)rapidity $y(\eta)$ can be calculated through
NLO QCD. In the case of gluon fusion, SusHi contains NLO QCD contributions from the
third family of quarks and squarks, NNLO corrections due to top-quarks, approximate
NNLO corrections due to top-squarks (for the MSSM), and electro-weak effects. It supports various
renormalization schemes for the sbottom sector and the bottom Yukawa coupling, as well as
resummation effects of higher order $\tan \beta$-enhanced sbottom contributions. SusHi provides
a link to FeynHiggs for the calculation of the MSSM Higgs masses.

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1. Introduction

The recent observation of a new boson at the Large Hadron Collider (LHC) \cite{2, 3} has opened a new chapter for Higgs phenomenology \cite{4}. For the clear identification of this particle, precise predictions for Higgs production and decay will be absolutely essential. The current status of these efforts is collected in the reports of the LHC Higgs cross section working group \cite{5, 6}.

The main production mechanism for a Standard Model (SM) Higgs boson at a hadron collider is gluon fusion, where the gluon-Higgs coupling is mediated mostly by virtual top- and bottom-quarks (\(\sim 7\%\)). The total inclusive cross section is known through next-to-next-to-leading order (NNLO) in quantum chromodynamics (QCD) \cite{7–18}. Even higher order QCD effects have been calculated \cite{19–24} through resummation; electro-weak contributions reach up to 8\% with respect to the leading order (LO) cross section \cite{25–27}.

The gluon-fusion mechanism for the neutral Higgs bosons in the Minimal Supersymmetric Standard Model (MSSM) is mediated by quarks and their superpartners, the squarks. For the CP-even MSSM Higgs bosons \(h, H\), the QCD effects due to quarks can be simply taken over from the SM by a rescaling of the cross section with the corresponding modified Yukawa couplings. For the CP-odd Higgs boson \(A\), the NNLO QCD corrections to the quark-induced total inclusive cross section have been calculated in Refs. \cite{18, 28, 29}. The same statement holds for the gluon-fusion mechanism in 2-Higgs-Doublet-Models (2HDM)
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[30], where a rescaling of the Yukawa coupling allows the inclusion of the third family quark QCD contributions to neutral Higgs production. For recent 2HDM reviews we refer to Refs. [31, 32].

Squark contributions to the gluon-Higgs coupling are typically suppressed by powers of \( m_q/m_{\tilde{q}} \), and thus of importance mostly for small to moderate squark masses \( m_{\tilde{q}} \). For the CP-odd Higgs boson \( A \), they are absent at LO [3].

The individual components for a calculation of NLO QCD corrections in the MSSM are known: The NLO cross section due to (s)top (s)quarks, gluons, and gluinos was calculated for a CP-even Higgs boson with mass \( m_\phi \) in Refs. [35–37] by applying an effective-theory approach in the limit \( m_\phi \ll m_t, m_\tilde{t}, m_\tilde{g} \), similar to what is used in the SM at higher orders. In this approach, even NNLO effects have been first approximated [38], and recently fully calculated [39, 40]. The analogous NLO result for a CP-odd Higgs was first obtained in Ref. [41]. Due to the smallness of the bottom-quark mass, these results cannot be transferred to the bottom/sbottom sector. However, the limit \( m_\phi, m_b \ll m_\tilde{b}, m_\tilde{g} \) is applicable in a large region of the parameter space, and the corresponding results were presented in Refs. [42, 43]. Recently, more general results for the cross sections, allowing for larger Higgs masses, were obtained for the CP-even and -odd Higgs bosons and both for the top/stop and the bottom/sbottom sector, in Refs. [44, 45]. A fully numerical calculation of the gluon-Higgs form factor for general quark/squark/gluino/Higgs-masses has been reported in Ref. [46]. For the pure squark contributions, the full Higgs-mass dependance of the NLO contribution to the cross section was presented (numerically and/or analytically) in Refs. [47–49].

For large values of the 2HDM/MSSM parameter \( \tan \beta \), the coupling of the light CP-even Higgs boson to bottom-quarks is significantly enhanced relative to the SM Yukawa coupling, so the bottom sector may be much more important for gluon fusion in the 2HDM/MSSM. In addition, an enhanced bottom-Higgs coupling increases the cross section of another Higgs production mechanism in the 2HDM/MSSM, namely associated production with bottom-quarks, \( pp/p\bar{p} \to b\bar{b}_\phi \). If the final state quarks are not tagged, a suitable theoretical approach to the cross section is the process \( b\bar{b} \to \phi \), called bottom-quark annihilation in what follows [2]. It resums terms of the form \( \ln m_b/m_\phi \) by means of b-parton distribution functions (PDFs) and was calculated up to NNLO QCD in the SM [51, 52]. The result can be directly translated into the MSSM by rescaling it with the proper bottom-Yukawa coupling; even the dominant sbottom effects can be taken into account by an effective coupling [53, 54]. SusHi evaluates both the cross section for gluon fusion and bottom-quark annihilation.

For gluon fusion, SusHi includes results for all NLO QCD contributions due to the third generation of quarks and squarks. The real corrections at NLO are well-known; SusHi implements them using the routines of Ref. [43]. For the virtual corrections to the pure quark diagrams, it uses the analytic expression of Ref. [55] which was obtained from the integral representation in Ref. [10]. Concerning the genuine virtual supersymmetric (SUSY) corrections, it employs the results of Refs. [35, 36, 41, 45] and Refs. [42, 44] for the (s)top- and the (s)bottom-mediated gluon-Higgs coupling, respectively. NNLO QCD effects are

1 Comprehensive reviews of the Higgs theory within the SM and the MSSM can be found in Refs. [33, 34].

2 See Ref. [50] for a more detailed discussion.
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taken into account for the top-quark induced gluon-Higgs coupling \[11,13,28,29\], and approximately for the top/stop/gluino-induced one \[38\] by using \texttt{ggh@nnlo} \[56\]. Electro-weak corrections \[23,27\] are included as tabulated correction factors. The cross section is provided in various renormalization schemes (in particular in the sbottom sector), allowing for an on-shell, DR, or a dependent renormalization of the soft-breaking parameter \(A_b\), for example. In addition, the bottom-Yukawa coupling can be chosen on-shell or in the \(\overline{\text{MS}}\) scheme. Higher-order sbottom effects can be included through the parameter \(\Delta_b\) \[57–62\].

For the calculation of the bottom-quark annihilation cross section, SusHi makes use of \texttt{bbh@nnlo} \[63\] and re-weights its results by the 2HDM/MSSM couplings. It uses the LHAPDF library \[64\] which allows to conveniently switch between different PDF sets, and it can be linked to \texttt{FeynHiggs} \[65–68\] for the two-loop calculation of the Higgs boson masses in the MSSM.

Apart from inclusive cross sections for gluon fusion and bottom-quark annihilation, SusHi allows for (upper and lower) cuts on the transverse momentum and/or the (pseudo-)rapidity of the outgoing scalar \(\phi\). In case of gluon fusion, differential distributions with respect to these kinematic variables can be obtained (for \(p_T\) not too small).

Note that a number of codes for the calculation of Higgs cross sections in the SM and the MSSM exist, see Refs. \[69–75\], for example. They overlap with SusHi to a greater or lesser extent; the distinctive feature of SusHi is to provide full NLO QCD (and partial NNLO and electro-weak) corrections for the dominant production mechanisms of the three neutral Higgs bosons of the 2HDM/MSSM, both inclusive and differential, in various renormalization schemes. Further details will be given below.

The remainder of this paper is organized as follows: In Section 2 we present the physical background of SusHi, recalling the framework of the Higgs and quark sectors in the 2HDM and the MSSM. Special emphasis is put on the renormalization of the MSSM squark sectors and the resummation of \(\tan \beta\)-enhanced sbottom corrections in the bottom Yukawa coupling. Subsequently, we discuss the various contributions for the calculation of the gluon-fusion and the bottom-quark annihilation cross section as they enter in SusHi. We briefly describe the kinematic variables for which cuts can be applied and distributions be obtained. In Section 6 we describe the program SusHi, in particular its workflow, installation, and usage, as well as the input and output files. Our conclusions are given in Section 7. Appendix A contains the couplings of the squarks to the Higgs bosons \(\phi\).

2. Physics background

This section first introduces our notation for the relevant parts of the SM and the MSSM. It describes the renormalization of the squark sector and the possible choices for the bottom Yukawa coupling provided in SusHi.

2.1. Standard Model

The SM contains one scalar weak isospin doublet which corresponds to a single physical particle \(H\) (electric charge and spin zero). The pure Higgs sector of the Lagrangian is determined by two parameters: the vacuum expectation value \(v \approx 246\, \text{GeV}\), and the mass
of the Higgs boson \( m_H \). The Yukawa couplings of the fermions to the Higgs boson are given by \( Y_f = \sqrt{2}m_f/v \), where \( m_f \) is the fermion mass. \texttt{SusHi} requires all input masses in the on-shell scheme, except for the charm- and bottom-quark mass which has to be given as the \( \overline{\text{MS}} \) mass \( m_q^{\overline{\text{MS}}} \), \( q \in \{c, b\} \). The calculation of various internal bottom masses is addressed in Section 2.4.

### 2.2. 2-Higgs-Doublet-Model

Two Higgs doublets \( H_1 \) and \( H_2 \) with vacuum expectation values \( v_1 \) and \( v_2 \) respectively are part of the 2HDM. Demanding CP conservation in the Higgs sector, no quartic terms in the Higgs doublets and no tree-level flavor-changing neutral currents, the 2HDM can be cast in four types, which differ by the Higgs-fermion Yukawa couplings. By convention \( H_2 \) couples to the right-handed up-type quarks, whereas the couplings to right-handed down-type quarks and leptons can be taken from Tab. 1. For more details we refer to Refs. [31, 32].

<table>
<thead>
<tr>
<th>Model</th>
<th>Type I</th>
<th>Type II</th>
<th>Lepton-specific</th>
<th>Flipped</th>
</tr>
</thead>
<tbody>
<tr>
<td>up-type quarks</td>
<td>( H_2 )</td>
<td>( H_2 )</td>
<td>( H_2 )</td>
<td>( H_2 )</td>
</tr>
<tr>
<td>down-type quarks</td>
<td>( H_2 )</td>
<td>( H_1 )</td>
<td>( H_2 )</td>
<td>( H_1 )</td>
</tr>
<tr>
<td>leptons</td>
<td>( H_2 )</td>
<td>( H_1 )</td>
<td>( H_1 )</td>
<td>( H_2 )</td>
</tr>
</tbody>
</table>

Table 1: 2HDM types implemented in \texttt{SusHi}.

The two Higgs doublets form two CP-even Higgs fields \( h \) and \( H \) and one CP-odd field \( A \). Their masses, the ratio of the vacuum expectation values \( \tan \beta = v_2/v_1 \) and the CP-even Higgs mixing angle \( \alpha \) are independent input parameters. The latter two angles determine the relative strength \( g_\phi^f (\phi \in \{h, H, A\}) \) of the Higgs-fermion Yukawa couplings \( Y_\phi^f = \sqrt{2}m_f g_\phi^f/v \), which are shown in Tab. 2 for the four different 2HDM types. The leptonic couplings are not presented, since they do not enter the cross section calculation. These normalized couplings are independent of the fermion generation.

<table>
<thead>
<tr>
<th>Model</th>
<th>Type I</th>
<th>Type II</th>
<th>Lepton-specific</th>
<th>Flipped</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_u^h )</td>
<td>( \cos \alpha/\sin \beta )</td>
<td>( \cos \alpha/\sin \beta )</td>
<td>( \cos \alpha/\sin \beta )</td>
<td>( \cos \alpha/\sin \beta )</td>
</tr>
<tr>
<td>( g_d^h )</td>
<td>( \cos \alpha/\sin \beta )</td>
<td>( -\sin \alpha/\cos \beta )</td>
<td>( \cos \alpha/\sin \beta )</td>
<td>( -\sin \alpha/\cos \beta )</td>
</tr>
<tr>
<td>( g_u^H )</td>
<td>( \sin \alpha/\sin \beta )</td>
<td>( \sin \alpha/\sin \beta )</td>
<td>( \sin \alpha/\sin \beta )</td>
<td>( \sin \alpha/\sin \beta )</td>
</tr>
<tr>
<td>( g_d^H )</td>
<td>( \sin \alpha/\sin \beta )</td>
<td>( \cos \alpha/\cos \beta )</td>
<td>( \sin \alpha/\sin \beta )</td>
<td>( \cos \alpha/\cos \beta )</td>
</tr>
<tr>
<td>( g_u^A )</td>
<td>( \cot \beta )</td>
<td>( \cot \beta )</td>
<td>( \cot \beta )</td>
<td>( \cot \beta )</td>
</tr>
<tr>
<td>( g_d^A )</td>
<td>( -\cot \beta )</td>
<td>( \tan \beta )</td>
<td>( -\cot \beta )</td>
<td>( \tan \beta )</td>
</tr>
</tbody>
</table>

Table 2: Relative couplings \( g_\phi^f \) with respect to the SM Yukawa coupling for the four 2HDM types.

### 2.3. Supersymmetry

The MSSM contains two Higgs doublets, named \( H_d \) and \( H_u \), which develop the vacuum expectation values \( v_d = v \cos \beta \) and \( v_u = v \sin \beta \), where the parameter \( \beta \) is undetermined.
Similar to the 2HDM they form two CP-even Higgs fields $h, H$, one CP-odd (or “pseudoscalar”) Higgs field $A$, and two charged Higgs fields $H^\pm$. At lowest order, the mass spectrum of the Higgs sector is determined by SM parameters, $\tan \beta = v_u/v_d$, and the CP-odd Higgs mass $m_A$. Radiative corrections to the Higgs mass spectrum are generally quite large \cite{76–78}; currently, they are known through three-loop order \cite{79–81}. \texttt{SusHi} \cite{65–68}. This is an optional feature, however; any Higgs mass can be given as an input to \texttt{SusHi} and coincide with the ones from Type II of the 2HDM in Tab.\cite{2}. Our calculation includes the third generation of squarks which enters the Lagrangian in the form

$$\mathcal{L} \supset -(\mathbf{1}_{4}^L, \mathbf{1}_{4}^R) \mathcal{M}_q^2 (\mathbf{1}_{4}^L, \mathbf{1}_{4}^R) ,$$

with the mass matrix

$$\mathcal{M}_q^2 = \begin{pmatrix} \mathcal{M}_L^2 + m_q^2 + m_Z^2 \cos(2\beta)(T_q^3 - Q_q s_W^2) & m_q(A_q - \mu k_q) \\ m_q(A_q - \mu k_q) & \mathcal{M}_R^2 + m_q^2 + m_Z^2 \cos(2\beta)Q_q s_W^2 \end{pmatrix}$$

for an arbitrary species of squarks \(\tilde{q}\). This formula contains the SUSY soft-breaking parameters \(M_L^2, M_R^2,\) and \(A_q\). The parameter \(\mu\) determines the mass of the fermionic Higgs partners, the Higgsinos, whereas the Z-boson mass \(m_Z\) and the weak mixing angle \(\theta_W\) (\(s_W = \sin \theta_W\)) are the usual SM parameters; \(m_q, Q_q,\) and \(T_q^3\) are the mass, the electric, and the weak charge of the corresponding quark \(q\), respectively. It is \(k_q = \tan \beta =: t_3\) and \(k_t = 1/t_3\).

The physical particle states are obtained by the diagonalization of the mass matrix in Eq. (2) which we do in accordance with Ref. \cite{82} using

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = U_\tilde{q} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} \quad \text{with} \quad U_\tilde{q} = \begin{pmatrix} \cos \theta_\tilde{q} & \sin \theta_\tilde{q} \\ -\sin \theta_\tilde{q} & \cos \theta_\tilde{q} \end{pmatrix} .$$

By choosing \(0 \leq \theta_\tilde{q} < \pi\), the masses of the squarks \(\tilde{q}_1\) and \(\tilde{q}_2\) are ordered \(m_{\tilde{q}_1} < m_{\tilde{q}_2}\) and given by the square roots of the eigenvalues of \(\mathcal{M}_q^2\) in Eq. (2):

$$m_{\tilde{q}_{12}}^2 = \frac{1}{2}(\mathcal{M}_L^2 + \mathcal{M}_R^2) + m_q^2 + \frac{1}{2}T_q^3 m_Z^2 \cos(2\beta) \pm \frac{1}{2}\sqrt{(\mathcal{M}_L^2 - \mathcal{M}_R^2 + m_Z^2 \cos(2\beta)(T_q^3 - 2Q_q s_W^2 \sin^2 \theta_W))^2 + 4m_q^2 (A_q - \mu k_q)^2} .$$

The entries of \(\mathcal{M}_q^2\) can also be expressed in terms of the mass eigenvalues and the mixing angle

$$\mathcal{M}_q^2 = \begin{pmatrix} m_{\tilde{q}_1} \cos^2 \theta_\tilde{q} + m_{\tilde{q}_2}^2 \sin^2 \theta_\tilde{q} & (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2) \sin \theta_\tilde{q} \cos \theta_\tilde{q} \\ (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2) \sin \theta_\tilde{q} \cos \theta_\tilde{q} & m_{\tilde{q}_1}^2 \sin^2 \theta_\tilde{q} + m_{\tilde{q}_2}^2 \cos^2 \theta_\tilde{q} \end{pmatrix} .$$
which implies that the mixing angle can be obtained from
\[
\sin(2\theta_q) = \frac{2m_q(A_q - \mu \kappa_q)}{m_{q1}^2 - m_{q2}^2}.
\] (6)

Note that Eq. (6) does not uniquely define \(\theta_q\); a shift \(\theta_q \rightarrow \frac{\pi}{2} - \theta_q\), which corresponds to \(\sin \theta_q \leftrightarrow \cos \theta_q\), might be in order to allow for \(m_{\tilde{q}1} < m_{\tilde{q}2}\). For completeness, the couplings of the squarks to the MSSM Higgs bosons can be found in Appendix A.

2.3.1. Renormalization of the (s)top sector

Using Eq. (6), we eliminate \(A_t\) from the (s)top sector contribution of the amplitude before renormalization and express it in terms of the on-shell parameters for the top mass \(m_t\), the stop masses \(m_{\tilde{t}1}\) and \(m_{\tilde{t}2}\) and the mixing angle \(\theta_t\), defined according to Section 3.1 in Ref. [82].

In practice, the user specifies the soft-breaking parameters \(M_L \equiv M_{L}(\tilde{t}), M_{R},\) and \(A_t\), as well as the on-shell top-quark mass \(m_t^{OS}\). Setting \(m_t = m_t^{OS}\), \(\text{SusHi}\) inserts them into the mass matrix of Eq. (2) whose eigenvalues \(m_{t1}^2, m_{t2}^2\) as well as the corresponding stop mixing angle \(\theta_t\) are interpreted as on-shell parameters. Note that in case \(\text{FeynHiggs}\) is used, the on-shell stop masses and the stop mixing angle are simply taken over from its output.

2.3.2. Renormalization of the (s)bottom sector

The renormalization of the (s)bottom sector is more subtle. At tree-level, the soft-breaking parameter \(M_B^2\) is identical for the sbottom and stop sector due to \(SU(2)_L\) symmetry. At higher orders, however, in the on-shell scheme we distinguish \(M_B^2(\tilde{t})\) and \(M_B^2(\tilde{b})\) similar to Ref. [83, 86]

\[
M_B^2(\tilde{b}) = M_B^2(\tilde{t}) + \delta M_B^2(\tilde{b}) \equiv M_B^2(\tilde{t}) + \Delta M_B^2,
\] (7)

with the individual counterterms given by

\[
\delta M_B^2(\tilde{q}) = \cos^2 \theta_q m_{\tilde{q}1}^{2,OS} + \sin^2 \theta_q m_{\tilde{q}2}^{2,OS} - (m_{\tilde{q}1}^2 - m_{\tilde{q}2}^2) \sin(2\theta_q) \delta \theta_q^{OS} - 2m_q \delta m_q^{OS},
\] (8)

where the on-shell counterterms \(\delta m_{\tilde{q}1}^{2,OS}, \delta m_{\tilde{q}2}^{2,OS}, \delta m_q^{OS}\) and \(\delta \theta_q^{OS}\) are analogously defined as in the top sector, see Ref. [82].

Note that the finite shift \(\Delta M_B^2\) depends on \(m_{\tilde{b}1}, m_{\tilde{b}2},\) and \(\theta_b\). In order to determine its numerical value, we first calculate “tree-level” values for the sbottom masses and mixing angle by inserting the parameters

\[
M_B^2 \equiv (M_B^2(\tilde{b}))^{tree} = M_B^2(\tilde{t}), \quad M_{bb}^2, \quad A_b, \quad m_b^{OS}
\] (9)

into the mass matrix (2) (setting \(m_b = m_b^{OS}\)). All parameters of Eq. (9) are input to \(\text{SusHi}\), except for the on-shell bottom-quark mass \(m_b^{OS}\) which is determined from the input parameter \(m_b(m_b)\) as described in Section 2.4. With these tree-level sbottom masses we define the scale

\[\text{This can be seen as an indirect definition of the renormalization scheme for } M_L, M_{LR}, \text{ and } A_t.\]
\[ \mu_a = \frac{1}{3}(m_\tilde{g} + m_{\tilde{b}_1} + m_{\tilde{b}_2}). \]  

(10)

As was pointed out in Refs. [42, 82, 87, 88], replacing \( A_b \) in the amplitude through Eq. (6) before renormalization – analogous to the stop sector – leads to potentially large corrections \( \delta A_b \propto (a_s \mu^2 \tan^2 \beta)/m_\tilde{g} \). It was therefore suggested to use Eq. (6) in order to eliminate \( m_b \). The counterterm for \( m_b \) in this scheme, denoted \( \delta m_b^{\text{dep}} \), is obtained from Eq. (6):

\[
\delta m_b^{\text{dep}} = 2m_b \cot(2\theta_b)\delta \theta_b - \frac{2m_b^2 \cdot \delta A_b}{\sin(2\theta_b)(m_{b_1}^2 - m_{b_2}^2)} + m_b \frac{\delta m_{b_1}^{\text{OS}} - \delta m_{b_2}^{\text{OS}}}{m_{b_1}^2 - m_{b_2}^2}.
\]  

(11)

Here it is already implied that we always renormalize the sbottom masses on-shell, while the renormalization of \( A_b \) and \( \theta_b \) is still unspecified.

In order to calculate the on-shell sbottom masses, we choose the “on-shell” renormalization of \( A_b \) defined through a kinematical condition on the \( Ab_1b_2 \)-vertex. Solving Eq. (11) (with \( \delta A_b = \delta A_b^{\text{OS}} \)) and Eq. (12) for \( \delta A_b^{\text{OS}} \), we obtain

\[
\delta A_b^{\text{OS}} = (A_b + \mu \cot \beta) \left[ f(m_{b_1}^2, m_{b_2}^2) + f(m_{b_2}^2, m_{b_1}^2) - \frac{\delta m_b^{\text{dep}}}{m_b} \right],
\]

(12)

with

\[
f(m_1^2, m_2^2) = -\frac{\alpha_s(\mu_a)}{\pi} \frac{2}{3} \left\{ \left[ -\frac{m_\tilde{g}}{A_b + \mu \cot \beta} B_0^{\text{fin}}(m_1^2, m_b, m_\tilde{g}, \mu_a) \right] + \frac{m_1^2}{m_1^2 - m_2^2} \left[ 4 + 2 \log \frac{m_R^2}{m_1^2} - (1 - \frac{m_\tilde{g}^2}{m_1^2} - \frac{m_b^2}{m_1^2}) \cdot B_0^{\text{fin}}(m_1^2, m_b, m_\tilde{g}, \mu_a) \right] \right\},
\]

(13)

where the function \( B_0^{\text{fin}} \) can be taken from Eq. (B.8) in Ref. [36] replacing \( \mu_R \rightarrow \mu_a \). Solving Eq. (11) (with \( \delta A_b = \delta A_b^{\text{OS}} \)) and Eq. (12) for \( \delta A_b^{\text{OS}} \), we obtain

\[
\delta A_b^{\text{OS}} = (A_b \sin \beta + \mu \cos \beta) \left\{ -(m_{b_1}^2 - m_{b_2}^2) \sin(2\theta_b) \left[ f(m_{b_1}^2, m_{b_2}^2) + f(m_{b_2}^2, m_{b_1}^2) \right] \right.
\]

\[ + \left. (\delta m_{b_1}^{\text{OS}} - \delta m_{b_2}^{\text{OS}}) \sin(2\theta_b) + 2\delta \theta_b (m_{b_1}^2 - m_{b_2}^2) \cos(2\theta_b) \right\} \]

\[ \cdot \left[ 2(\sin \beta (A_b m_b - (m_{b_1}^2 - m_{b_2}^2) \sin \theta_b \cos \theta_b) + m_b \mu \cos \beta) \right]^{-1}, \]

(14)

which in turn allows to calculate \( \delta m_b^{\text{dep}} \). Note that one is still free to choose the renormalization condition for \( \theta_b \).

The on-shell sbottom masses are finally obtained as follows: We calculate the counterterm \( \delta m_b^{\text{dep}} \) from Eq. (11) using \( \delta A_b = \delta A_b^{\text{OS}} \) and \( \delta \theta_b = \delta \theta_b^{\text{OS}} \), the “tree-level” sbottom masses and mixing angle and the on-shell bottom mass. This yields a numerical value for \( \Delta M_b^2 \) and similarly for \( m_b^{\text{dep}} = m_b^{\text{DR}}(\mu_a) - \delta m_b^{\text{dep}} + \delta m_b^{\text{DR}} \), where the \( \text{DR} \) mass is calculated in the SM. Diagonalizing the mass matrix \( M_b^2 \) in Eq. (2) with \( m_b = m_b^{\text{dep}} \), we thus obtain on-shell sbottom masses \( m_{b_1} \) and \( m_{b_2} \) and the on-shell sbottom mixing angle \( \theta_b \). In case FeynHiggs is called for the calculation of the MSSM Higgs masses, we take \( m_b^{\text{dep}} \) as well as \( \Delta M_b^2 \) from Manual SusHi 1.1.0
this program which, when inserted into the sbottom mass matrix Eq. (2), results in on-shell sbottom masses and mixing angle consistent with the values given by FeynHiggs itself.

After the on-shell sbottom masses have been determined, SusHi allows for a change between various schemes regarding the renormalization of $m_b$, $A_b$, and $\theta_\tilde{b}$ in the sbottom contribution to the gluon-fusion amplitude. Note, however, that either $m_b$ or $A_b$ is required to be a dependent quantity; a dependent renormalization of the sbottom mixing angle is not offered as option. The possible choices are summarized in Table 3. Numerical differences in the cross sections between the various renormalization schemes and their implications will be investigated in a separate publication.

<table>
<thead>
<tr>
<th>Scheme choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_b$</td>
</tr>
<tr>
<td>$A_b$</td>
</tr>
<tr>
<td>$\theta_\tilde{b}$</td>
</tr>
</tbody>
</table>

Table 3: Available renormalization schemes for the sbottom sector. The default option is marked in red.

To summarize, apart from $\mu$, $\tan \beta$ and the SM parameters, the input parameters of SusHi that determine the squark sectors are:

- $M^2_{L_{t_R}}, M^2_{t_R}, A_t, m_t^{OS}$ which directly determine the on-shell stop masses and mixing angle through diagonalization of Eq. (2)

- $M^2_{b_{L_R}}, A_b, m_b(m_b)$, where $A_b$ is understood as renormalized according to Eq. (12).

Switching between renormalization schemes changes both the counterterms to the amplitude as well as the numerical values of the parameters $m_b$, $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $A_b$, and $\theta_\tilde{b}$, of course. We remark that changing the renormalization scheme in the sbottom sector affects $m_b$ only in the Higgs-sbottom couplings; the renormalization of the bottom mass occurring in the Higgs-bottom Yukawa coupling is independent of that (see Section 2.4). For the bottom mass occurring in internal propagators (rather than in couplings), SusHi always uses the on-shell value.

The switching between the different renormalization schemes is done at the renormalization scale $\mu_R$ to guarantee the same coupling strength $\alpha_s(\mu_R)$ for the NLO counterterms and the cross section itself. In case SusHi makes use of the formulas in Refs. [42, 44], the counterterms at NLO are expanded to the correct order in the bottom mass to match the expanded NLO amplitudes.

2.4. Bottom mass calculation/Resummation of $\tan \beta$-enhanced contributions

The bottom-quark mass in the input files of SusHi is inserted in the $\overline{MS}$ scheme $m_b(m_b)$. Together with the input value of $\alpha_s(m_Z)$, we calculate $\alpha_s(m_b(m_b))$ by 4-loop running with 5 active flavors. Using Eq. (13) of Ref. [89] at 3-loop level (see also Ref. [90]), $m_b(m_b)$ is transformed into its on-shell value $m_b^{OS}$. 


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In the SM, the bottom Yukawa coupling $Y_b = \sqrt{2} m_Y / v$ can be chosen $m_Y = m_{\text{OS}}$ or alternatively $m_Y = m_{\text{MS}}(\mu_b)$, where $\mu_b \in \{m_b, \mu_R\}$; $\mu_R$ denotes the renormalization scale. As indicated above, the bottom mass entering the internal propagators is always set to the on-shell mass $m_{\text{OS}}$ in SusHi. In the MSSM, SusHi offers various options regarding the choice of the Higgs-bottom Yukawa coupling $Y_{b} = \sqrt{2} m_{\phi} g_{\phi} / v$:

- on-shell coupling: $m_{b,\phi} = m_{\text{OS}}$ (15a)
- basic resummation: $m_{b,\phi} = \frac{m_{\text{OS}}}{1 + \Delta_b}$ (15b)
- full resummation $h$: $m_{b,h} = \frac{m_{\text{OS}}}{1 + \Delta_b} \left(1 - \frac{\Delta_b \cot \alpha}{\tan \beta}\right)$ (15c)
- full resummation $H$: $m_{b,H} = \frac{m_{\text{OS}}}{1 + \Delta_b} \left(1 + \frac{\Delta_b \tan \alpha}{\tan \beta}\right)$ (15d)
- full resummation $A$: $m_{b,A} = \frac{m_{\text{OS}}}{1 + \Delta_b} \left(1 - \Delta_b \frac{1}{\tan^2 \beta}\right)$ (15e)
- Running coupling: $m_{b,\phi} = \frac{m_{\text{MS}}(\mu_b)}{1 + \Delta_b}$ (15f)

Herein, $\Delta_b$ resums higher order sbottom contributions as shown in Fig. 1 [57–62], for example, calculated at the scale $\mu_d$, defined in Eq. (10). The exact formula is given by:

$$\Delta_b = \frac{2}{3\pi} \alpha_s(\mu_d) m_\beta \mu_\beta I(m_{b_1}^2, m_{b_2}^2, m_{\tilde{g}}^2)$$ (16)

$$I(a, b, c) = \frac{ab \ln \left(\frac{a}{b}\right) + bc \ln \left(\frac{b}{c}\right) + ca \ln \left(\frac{c}{a}\right)}{(a - b)(b - c)(a - c)}$$ (17)

If the input values for SusHi are determined by FeynHiggs, SusHi allows to use the value of $\Delta_b$ as given in the output of this program which contains also the electro-weak contributions from neutralinos and charginos in accordance with Ref. [91]; two-loop corrections to $\Delta_b$ [92–94] are not yet included. For the running coupling in Eq. (15f), the scale $\mu_b$ can be set to $m_b$ or $\mu_R$. The numerical differences between the various schemes will be discussed in a forthcoming publication.

3. Cross section for gluon fusion

As indicated in the Introduction, the most important production channel in the SM and for moderate values of $t_\beta$ in the 2HDM/MSSM is gluon fusion. After quoting the well-known results for the LO cross section, our implementation of the NLO contributions is explained. A discussion of NNLO and electro-weak contributions follows.
Using the notation of Ref. [10], the hadronic cross section for \( \phi \in \{ h, H, A \} \) at NLO QCD can be written as follows:

\[
\sigma(pp \rightarrow \phi + X) = \sigma^\phi_0 \left[ 1 + C^\phi \frac{\alpha_s}{\pi} \tau_\phi \frac{dL^gg}{d\tau_\phi} + \Delta\sigma^\phi_{gg} + \Delta\sigma^\phi_{gq} + \Delta\sigma^\phi_{qq}, \right. (18)
\]

where \( \tau_\phi = m_\phi^2/s \), with the hadronic center-of-mass energy \( s \). The factor \( \sigma^\phi_0 \) is determined by the LO cross section, \( C^\phi \) arises from NLO terms in the partonic cross section that are singular as \( \hat{s} \rightarrow m_\phi^2 \) (\( \hat{s} \) is the partonic center-of-mass energy), and

\[
\frac{dL^gg}{d\tau} = \int_1^\infty \frac{dx}{x} g(x)g(\tau/x) (19)
\]

is the gluon-gluon luminosity. The quantities \( \Delta\sigma^\phi_{gg}, \Delta\sigma^\phi_{gq}, \) and \( \Delta\sigma^\phi_{qq} \) comprise the terms that are regular as \( \hat{s} \rightarrow m_\phi^2 \) in the partonic cross section, arising from \( gg, gq \) and \( qq \) scattering, respectively. Loosely speaking, \( C^\phi \) is due to the virtual, while the \( \Delta\sigma^\phi_{ij} \) are due to the real radiation contributions. The latter are implemented in \texttt{SusHi} by expressing them in terms of Passarino-Veltman functions [95], see Ref. [43]. The strong coupling \( \alpha_s \) for the cross section calculations is renormalized in standard QCD with five active quark flavors.

3.1. LO cross section

Taking into account the third generation of quarks (and squarks in the MSSM), the normalization factor in Eq. (18) can be written in the form

\[
\sigma^\phi_0 = \frac{G_F \alpha_s^2(\mu_R)}{288 \sqrt{2} \pi} |A^{\phi,(0)}|^2, \quad (20)
\]

with Fermi’s constant \( G_F \). The amplitude \( A \) for is given by

\[
A^{\phi,(0)} = \sum_{q \in \{ t, b \}} \left( a^{\phi,(0)}_q + \tilde{a}^{\phi,(0)}_q \right), \quad (21)
\]

with the individual contributions for \( \phi \in \{ h, H \} \)

\[
a^{\phi,(0)}_q = g^\phi_q \frac{3\tau^\phi_q}{2} \left( 1 + (1 - \tau^\phi_q)f(\tau^\phi_q) \right), \quad \tilde{a}^{\phi,(0)}_q = -\frac{3\tau^\phi_q}{8} \sum_{i=1}^2 g^\phi_{qi,i} \left( 1 - \tau^\phi_q f(\tau^\phi_{qi}) \right), \quad (22)
\]

using the notation

\[
\tau^\phi_q = \frac{4m_q^2}{m_\phi^2}, \quad \tau^\phi_{qi} = \frac{4m_{\tilde{q}_i}^2}{m_\phi^2} \quad (23)
\]

and the function

\[
f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left( \log \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right)^2 & \tau < 1 \end{cases} \quad (24)
\]
For the CP-odd Higgs $\phi = A$, the squarks do not contribute at LO, i.e. $\tilde{a}^{A,(0)}_q = 0$, and the quark contribution can be written in the form

$$a^{A,(0)}_q = g^A_q \frac{3\tau^A_q}{2} \tau^A_q f(\tau^A_q).$$  \hspace{1cm} (25)$$

The couplings $g^\phi_q$ of the Higgs $\phi$ to the quarks can be taken from Tab. 2 (Type II for MSSM), the couplings $g^\phi_{\tilde{q},ij}$ to the squarks from Appendix A. Needless to say, in the SM, the squark couplings have to be set to zero (also for the 2HDM) and the quark couplings to $g^\phi_q = 1$.

### 3.2. NLO virtual contributions

As indicated above, the coefficient $C^\phi$ contains the virtual corrections to the $gg$ process and is regularized by the infrared singular part; moreover, it includes the counterterms to LO quantities. We write it as

$$C^\phi = 2\text{Re} \left[ \frac{A^{\phi,(1)}_{\infty}}{A^{\phi,(0)}_{\infty}} \right] + \pi^2 + \beta_0 \log \left( \frac{\mu^2_R}{\mu^2_F} \right),$$  \hspace{1cm} (26)$$

where $\beta_0 = 11/2 - n_f/3$ with $n_f = 5$; $\mu_F$ and $\mu_R$ denote the factorization and the renormalization scale, respectively. The NLO amplitude $A^{\phi,(1)}_{\infty}$ and the LO amplitude $A^{\phi,(0)}_{\infty}$ in the limit of large stop and sbottom masses are given by

$$A^{\phi,(1)}_{\infty} = \sum_{q \in \{t,b\}} (a^{\phi,(1)}_q + \tilde{a}^{\phi,(1)}_q), \quad A^{\phi,(0)}_{\infty} = \sum_{q \in \{t,b\}} \left( a^{\phi,(0)}_q + \frac{\tau^q_\phi}{8} \sum_{i=1}^{2} \frac{g^\phi_{\tilde{q},ii}}{\tau^\phi_{\tilde{q},i}} \right).$$  \hspace{1cm} (27)$$

Available results for the NLO contributions have been discussed in the Introduction. In SusHi, we use the analytic formulas of Ref. [55] for the quark-induced terms $a^{\phi,(1)}_q$. The purely squark-induced terms (see Fig. 2(a), for example) need to be considered in combination with the mixed quark/squark/gluino diagrams (two examples are shown in Fig. 2) in order to preserve supersymmetry, resulting in the coefficient $\tilde{a}^{\phi,(1)}_q$. SusHi implements expansions for these amplitudes in two limits:

- $m_\phi \ll m_q, m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}$ \cite{35,37} which is valid through $m_\phi < \min(2m_q, 2m_{\tilde{q}}, m_{\tilde{q}} + m_{\tilde{g}})$ and thus applies to the top-stop sector as long as $\phi$ is not too heavy. SusHi incorporates the publicly available program evalcsusy.f \cite{96} in order to use this result for the light Higgs $h$.

- $m_\phi, m_q \ll m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}$ \cite{42,45} which holds through $m_\phi < \min(2m_{\tilde{q}}, m_{\tilde{q}} + m_{\tilde{g}})$, and thus applies to the bottom-sbottom sector, as well as to the top-stop sector when $\phi$ is heavy; SusHi uses the formulas of Refs. \cite{44,45} in these cases.

Note that both expansions hold only as long as the Higgs mass is not much heavier than the typical SUSY mass. For larger Higgs masses, only the fully numerical result of Ref. \cite{46} is known so far.
3.3. NNLO corrections

While the implementation of NLO corrections allows for the evaluation of inclusive and exclusive quantities, NNLO corrections are available in SusHi only for the total inclusive cross section. In addition, only the top-(s)quark induced gluon-Higgs coupling is taken into account at NNLO; the top-squark induced one only approximately:

\[
\sigma_{g\phi,\text{NNLO}}^{\text{MSSM}} = \sigma_{g\phi,\text{NLO}}^{\text{MSSM}} + (\sigma_{t\phi,\text{NNLO}}^{t} - \sigma_{t\phi,\text{NLO}}^{t}),
\]

where, on the right-hand side, the NNLO term is evaluated with the NNLO PDFs, while the NLO terms are evaluated as usual with NLO PDFs.

The (N)NLO top-(s)quark contributions to the cross sections \(\sigma_{g\phi,\text{NNLO}}^{t}\) are calculated with the help of the programs \texttt{ggh@nnlo}\[56\] and \texttt{evalcsusy}\[96\] which work in the effective theory approach of heavy top (s)quarks. The NNLO top-squark (and mixed top/stop/gluino) effects, available for \(\phi = h\) only, have been evaluated\[39,40\] by applying the limit \(m_\phi \ll m_q, m_{\tilde{q}_1}, m_{\tilde{q}_2}, m_{\tilde{g}}\). The result has been implemented in a computer code that involves \texttt{Mathematica} and a number of other programs. SusHi includes an approximation of these NNLO effects according to Ref.\[38\].

3.4. Electro-weak corrections

The full NLO electro-weak (EW) corrections are known only in the SM\[25\]. It has been suggested to assume complete factorization of QCD and EW effects\[97\], thus writing

\[
\sigma_{g\phi,\text{NNLO,EW}}^{\text{SM,t}} = (1 + \delta_{\text{EW}})\sigma_{g\phi,\text{NNLO}}^{\text{SM,t}}.
\]

For the CP-even Higgs bosons in the MSSM, a formula based on Eq.\[28\] and Eq.\[29\] has been used for the combination of QCD and electro-weak corrections in Ref.\[43\]:

\[
\sigma_{g\phi,\text{NNLO,EW}}^{2\text{HDM,MSSM}} = \sigma_{g\phi,\text{NLO}}^{2\text{HDM,MSSM}} + (1 + \delta_{\text{EW}})\sigma_{g\phi,\text{NNLO}}^{t} - \sigma_{g\phi,\text{NLO}}^{t},
\]

\footnote{The NNLO term of the normalized Wilson coefficient \((\kappa_2\text{ in the notation of Ref.}\[38\])\) is replaced by its SM value in SusHi.}
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or, at NLO precision,

\[ \sigma_{2\text{HDM,MSSM}}^{2HDM,MSSM} = \sigma_{2\text{HDM,MSSM}}^{2HDM,MSSM} + \delta_{\text{EW}} \sigma_{2HDM,MSSM}^{NLO} \cdot \]

Alternatively, it has been suggested in Ref. [70] to use the SM electro-weak corrections due to light quarks only [26, 27]. Following Ref. [70], we define the correction factor

\[ \delta_{\text{lf}}^{\text{EW}} = \frac{\alpha_{\text{EM}} \pi}{2 \text{Re} (A_{\Phi}^{(0)} A_{\Phi,\text{EW}}^{*})} \]

where \( A_{\Phi}^{(0)} \) denotes the complete LO amplitude including quark and squark diagrams, see Eq. (21), and the electro-weak amplitude is given by [27]

\[ A_{\Phi,\text{EW}} = - \frac{3x_{W}}{8s_{W}^{2}} \left( \frac{2}{3} - \frac{7}{3}s_{W}^{2} + \frac{22}{9}s_{W}^{4} \right) A_{1}[x_{Z}] + 4A_{1}[x_{W}] \]

with

\[ x_{V} = \frac{1}{m_{\phi}^{2}} \left( m_{V} - \frac{i\Gamma_{V}}{2} \right)^{2}, \quad V \in \{ W, Z \} \]

the electro-magnetic coupling \( \alpha_{\text{EM}} \), and \( s_{W} = \sin \theta_{W} = (1 - c_{W}^{2})^{1/2} \) the sine of the weak mixing angle. The Higgs mixing angle \( \alpha \) and the ratio \( \tan \beta \) in the 2HDM and the MSSM enter through the relative couplings \( g_{V}^{\phi} \) given by

\[ g_{V}^{h} = \sin(\beta - \alpha), \quad g_{V}^{A} = 0, \quad g_{V}^{H} = \cos(\beta - \alpha). \]

The function \( A_{1}[x] \) can be found in Refs. [26, 27]. Since its numerical evaluation is rather involved, SusHi implements \( \delta_{\text{EW}}^{\text{lf}} \) (and \( \delta_{\text{EW}}^{\text{lf}} \)) in terms of an interpolation grid in \( m_{\phi} \) (and \( m_{t} \)), using fixed values for the gauge boson masses and widths, as well as for the weak mixing angle \( \langle m_{W} = 80.385 \text{ GeV}, \Gamma_{W} = 2.085 \text{ GeV}, \sin^{2} \theta_{W} = 0.22295, m_{Z} = 91.1876 \text{ GeV}, \Gamma_{Z} = 2.4952 \text{ GeV} \rangle \); the input values to SusHi for these parameters are ignored in the evaluation of the electro-weak corrections. The electro-weak correction factor due to light quarks \( \delta_{\text{EW}}^{\text{lf}} \) multiplies the NLO 2HDM/MSSM cross section, while the NNLO QCD effects are simply added as in Eq. (28):

\[ \sigma_{2\text{HDM,MSSM}}^{2\text{HDM,MSSM}} = \sigma_{2\text{HDM,MSSM}}^{2\text{HDM,MSSM}} (1 + \delta_{\text{EW}}^{\text{lf}}) + \sigma_{gg\phi,\text{NNLO}}^{t} - \sigma_{gg\phi,\text{NLO}}^{t} \cdot \]

SusHi leaves it up to the user to decide whether to use Eq. (30) or Eq. (36) in order to include the electro-weak corrections. For a SM-like Higgs and \( m_{\phi} < 2m_{t} \), both approaches lead to comparable NLO results. The EW corrections for a CP-odd Higgs are not known and thus not included.
4. Cross section for bottom-quark annihilation

In supersymmetric theories, where the Higgs coupling to bottom-quarks can be enhanced by $\tan\beta$, associated production $(b\bar{b})\phi + X$ can be similarly or even more important than gluon fusion. Two theoretical approaches have been pursued for the theoretical description of this process: In the four-flavor scheme (4FS), the relevant production processes at lowest order QCD are $gg \to (b\bar{b})\phi$ (see Fig. 3(a)) and quark-antiquark annihilation $q\bar{q} \to (b\bar{b})\phi$ [99–101]. However, when integrating over all final-state bottom-quark momenta, potentially large logarithms $\ln m_b/m_{\phi}$ occur. They can be resummed by the introduction of bottom-quark PDFs, which defines the five-flavor scheme (5FS) [102, 103]. The LO process in this latter scheme is bottom-quark annihilation $b\bar{b} \to \phi$ for which the lowest-order Feynman diagram is shown in Fig. 3(b).

SusHi implements results for associated $b\bar{b}\phi$-production in the 5FS. For the inclusive cross section, it links the program $bbh@nnlo$ [52] in order to obtain the NNLO QCD prediction $\sigma_{bbH}^{SM}$ using $m_{b,\overline{MS}}(\mu_R)$ for the bottom Yukawa coupling. This is then re-weighted by the corresponding resummed SUSY coupling $\tilde{g}_b$ [61, 104] as follows

$$\sigma_{bb\phi}^{MSSM} = \sigma_{bbH}^{SM} \cdot (\tilde{g}_b^2)^2 \quad \text{with} \quad \tilde{g}_b = \frac{g_b^h}{1 + \Delta_b} \left(1 - \Delta_b \frac{\cot\alpha}{\tan\beta}\right), \quad (37)$$

$$\tilde{g}_b^H = \frac{g_b^H}{1 + \Delta_b} \left(1 + \Delta_b \frac{\tan\alpha}{\tan\beta}\right), \quad \tilde{g}_b^A = \frac{g_b^A}{1 + \Delta_b} \left(1 - \Delta_b \frac{1}{\tan^2\beta}\right), \quad (38)$$

where the $\tilde{g}_b^\phi$ are given in Tab. 2 and $\Delta_b$ is determined as described in Section 2.4. In case of the 2HDM the cross section is obtained by $\sigma_{bb\phi}^{2HDM} = \sigma_{bbH}^{SM} \cdot (\tilde{g}_b^\phi)^2$.

For differential cross sections due to bottom-quark annihilation, SusHi includes the NLO virtual corrections for $b\bar{b} \to \phi$ and combines them with the LO real-radiation processes $b\bar{b} \to g\phi$ and $bg \to b\phi$ using dipole subtraction. Similar to the fully inclusive case, we multiply with the resummed SUSY couplings to obtain MSSM cross sections.

\footnote{We are grateful to M. Wiesemann for providing us with the corresponding Fortran routines which entered the studies presented in Refs. 105, 106.}
5. Differential cross sections

Apart from the total inclusive cross sections due to gluon fusion and bottom-quark annihilation, SusHi also allows for the computation of differential cross sections in these processes. In particular, one may apply upper and lower cuts on the Higgs transverse momentum $p_T$, its rapidity $y$ or its pseudo-rapidity $\eta$, where

$$\eta = -\ln \left( \tan \frac{\theta}{2} \right) = \frac{1}{2} \left( \frac{\vec{p} + p_L}{|\vec{p} - p_L|} \right), \quad y = \frac{1}{2} \left( \frac{E + p_L}{E - p_L} \right).$$

(39)

Here, $\vec{p} = \vec{p}_T + p_L$ is the 3-momentum of the Higgs boson, $p_L$ the longitudinal component, $E$ the Higgs boson’s energy, and $\theta$ the scattering angle (all in the hadronic reference frame). For gluon fusion, SusHi also provides the differential quantities $d\sigma/dp_T$, $d\sigma/dy$, and $d^2\sigma/(dp_Tdy)$ (or, alternatively, $dy$ instead of $dy$). We add that, since the distribution in $y$ and $\eta$ is symmetric, minimal and maximal values for $y$ are understood as $0 \leq y_{\text{min}} \leq |y| \leq y_{\text{max}}$ (and similarly for $\eta$). In order to get reliable results, the precision for the numerical integration in SusHi should be set to a higher value for differential quantities than for inclusive cross sections. Some combinations of distributions and cuts are not allowed due to numerical instabilities.

Note that at LO, i.e., $O(\alpha_s^2)$ for gluon fusion and $O(\alpha_s^0)$ for bottom-quark annihilation, the Higgs transverse momentum is always $p_T = 0$. SusHi provides results for non-inclusive quantities through NLO, i.e., $O(\alpha_s^2)$ for gluon fusion and $O(\alpha_s)$ for bottom-quark annihilation. Let us also add that $p_T$-cuts or $p_T$-distributions should not be too low ($p_T/m_\phi \gtrsim 0.1$), since otherwise potentially large logarithms may spoil the perturbative convergence of the fixed-order results implemented in SusHi. For the resummation of such terms in Higgs production, see Refs. [107, 108], for example. The electroweak contributions by light quarks for gluon fusion can be added in case of distributions and cuts following Eq. (36).

6. The program SusHi

This section describes the most important technical details of the program SusHi, including its installation and usage.

6.1. Workflow

The workflow of SusHi is depicted in Fig.4. The input is controlled by a single input file whose format is SLHA-inspired [109, 110]. In case of the MSSM, the user specifies whether the Higgs mass is calculated by FeynHiggs or provided by the user himself. After the initialization of internal parameters which are derived from the input data, SusHi transforms them to the specified renormalization scheme and determines the resummation of $\tan \beta$-enhanced terms in the bottom Yukawa coupling, see Sections 2.3.2 and 2.4. Afterwards, the gluon fusion and bottom-quark annihilation cross sections are calculated up to the desired perturbative order. The NNLO top-(s)quark induced and the electro-weak contributions for gluon fusion are taken into account only for the inclusive cross section. Not shown in the workflow is the link to LHAPDF which occurs at various stages of the internal calculation. The output is printed to the screen and written to an output file which follows the same format as the input file. Details concerning the in- and output files are given in Section 6.4.
Figure 4: Internal workflow of SusHi. Red boxes indicate interaction with the user, who has to provide an input and gets an output file, if no error messages are shown. Green boxes refer to external code (see text), which is linked to/included in SusHi.

6.2. External code

As already mentioned, SusHi includes existing code like ggh@nnlo, bbh@nnlo, or the electro-weak grid. The integration of these programs does not require any action from the user though; they are part of the distribution and are simply linked to SusHi upon compilation.

However, SusHi can/must be linked to the following external code:

- **FeynHiggs** \[65\]-\[68\]: For the calculation of the supersymmetric Higgs masses $\phi \in \{h, H, A\}$, SusHi can be linked to FeynHiggs. Its input is controlled via the SusHi input files. Note that the current version of SusHi does not support complex MSSM parameters, so FeynHiggs is called with the default flags for the real MSSM.

- **LHAPDF** \[64\]: SusHi has to be linked to LHAPDF which provides a large variety of different PDF sets. This allows one to change the PDF set used by SusHi simply by changing the input file.
6.3. Installation and Usage

A tarball with the source files of SusHi can be obtained from Ref. [1]. Unpacking results in a main folder with the following subfolders:

bin : contains the executable program sushi after compilation

example : various example input files to be used in bin

include, lib : object files and libraries

src : SusHi source files, including external code

The file README in the main directory contains installation instructions and the history of the code. Compilation is most easily done by adjusting and running the Makefile in the main folder as follows:

- Specify the location and the name of the library of LHAPDF, for example:
  PDFLIBP = /usr/local/lib
  PDFLIB = -lLHAPDF

- If FeynHiggs should be used, specify the main directory of the compiled FeynHiggs library, for example:
  FHPATH = /home/.../FeynHiggs-x.x.x
  Note that SusHi requires FeynHiggs version 2.9 or higher.

- Run the configure script in the main folder:
  ```
  ./configure
  ```
  It tries to find the gfortran or ifort compiler and the dependences on your local machine. If you prefer a different compiler, or if the script fails, you can specify the relevant variables (F77 and LDFLAGS) in the file compilerissues yourself.

- In the main folder, run make:
  ```
  make [option]
  ```
  This command takes either of two optional arguments: make predef=FH includes the link to FeynHiggs (in contrast to previous versions SusHi ≤ 1.0.6, where FeynHiggs was included by default); make clean deletes all object files, libraries, and the executable sushi.

After compilation, one may perform a test run of SusHi by copying one of the input files from the example- to the bin-folder, change to the bin-folder, and run

```
./sushi sushi.in sushi.out
```

Note that apart from the input filename, the user also has to provide a name for the output file. For parameter scans, we recommend the auxiliary routines SLHAroutines, which can be downloaded from Ref. [11]. In the following we will discuss the input and output files in more detail.
6.4. Input and output files

SusHi allows for calculations in the SM as well as in the MSSM. Although the former is in many respects a limiting case of the latter, SusHi distinguishes both cases, and we will discuss them separately in what follows.

6.4.1. Standard Model

A typical input file for a SM calculation is shown in the following. It divides into blocks, each of which contains a number of entries, specified by one or more leading blanks, an integer “entry number”, a value, and a comment initiated by the hash symbol #.

Block SUSHI
1 0 # model: 0 = SM, 1 = MSSM, 2 = 2HDM
2 0 # 0 = scalar Higgs (h), 1 = pseudoscalar Higgs (A)
3 0 # collider: 0 = p-p, 1 = p-pbar
4 8000.0 # center-of-mass energy in GeV
5 2 # order ggh: -1 = off, 0 = LO, 1 = NLO, 2 = NNLO
6 2 # order bbh: -1 = off, 0 = LO, 1 = NLO, 2 = NNLO
7 2 # electroweak cont. for ggh:
8 0 = no, 1 = light quarks at NLO, 2 = SM EW factor

Block SMINPUTS # Standard Model inputs
1 1.27934000e+02 # alpha_em^(−1)(MZ) SM MSbar
2 1.16637000e-05 # G_Fermi
3 1.17200000e-01 # alpha_s(MZ) SM MSbar
4 9.11876000e+01 # m_Z (pole)
5 4.20000000e+00 # m_b (m_b)
6 1.73300000e+02 # m_t (pole)
7 1.27500000e+00 # m_c (m_c)

Block MASS # Higgs mass
1 125.000000d0 # Higgs mass

Block DISTRIB # distribution:
1 0 # pt-cut: 0 = no, 1 = pt > ptmin, 2 = pt < ptmax,
2 0 # 3 = ptmin < pt < ptmax
21 30.0 d0 # minimal pt-value ptmin in GeV
22 100.0 d0 # maximal pt-value ptmax in GeV
3 0 # rapidity-cut: 0 = no, 1 = Abs[y] < ymax,
31 0.5 d0 # minimal rapidity ymin
32 1.5 d0 # maximal rapidity ymax
4 0 # 0 = rapidity, 1 = pseudorapidity

Block SCALES # renormalization scale μR/mh
1 1.0 # factorization scale μF/mh
3 0 # 1 = Use (μR,μF)/Sqrt(mh^2+pt^2) for dsigma/dpt and d^2sigma/dy/dpt

Block RENORMBTO # Renormalization of the bottom sector
1 0 # m_b used for bottom Yukaws: 0 = OS, 1 = MSbar(mb), 2 = MSbar(muR)

Block PDFSPEC # name of pdf
1 MSTW2008lo68cl.LHgrid
2 MSTW2008nlo68cl.LHgrid
3 MSTW2008nnlo_asmzrange.LHgrid
4 0 # set number

Block VEGAS # number of points
1 10000 # number of iterations
3 10 # print: 0 = no output, 1 = prettyprint, 10 = table

Block FACTORS # factor for yukawa-couplings:
1 0.d0 # c
2 1.d0 # t
3 1.d0 # b

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Block SUSHI specifies the crucial input for SusHi, namely the model, the kind of Higgs boson to be considered (scalar or pseudo-scalar\textsuperscript{6}), the type of collider, the center-of-mass energy, the perturbative order for gluon fusion and bottom-quark annihilation, and to which extent electro-weak corrections to gluon fusion should be taken into account.

Block SMINPUTS contains the relevant SM input. We use the electro-magnetic coupling $\alpha_{\text{EM}}$, Fermi’s constant $G_F$, and the $Z$-boson mass $m_Z$ (entries 1,2,4) to calculate the $W$ mass $m_W$ and the weak mixing angle $\sin \theta_W$. The input value for $\alpha_s(m_Z)$ given in entry 3 is used for renormalization-group (RG) running and RG transformations. We allow this value to be different from the one required by the PDFs which are specified further below. The latter is taken from LHAPDF and enters the calculation as the coupling parameter of the perturbative expansion of the cross section, for example Eq. (18) and Eqs. (20). The charm- and bottom-quark masses (entries 8,5) are to be given in the $\overline{\text{MS}}$ scheme as $m_c(m_c)$ and $m_b(m_b)$, while the top-quark mass (entry 6) is required in the on-shell scheme. In the SM, the Higgs mass is a free parameter and has to be provided in Block MASS, entry 1.

Block DISTRIB controls cuts or distributions with respect to the transverse momentum $p_T$, the (pseudo-)rapidity $y$ ($\eta$), if desired. Note that differential cross sections (entry 1 $\in \{1, 2, 3\}$) can only be obtained for gluon fusion; in this case, entries 22 and/or 32 specify the value of $p_T$ and/or $y$ ($\eta$). Cuts are possible both for gluon fusion and bottom-quark annihilation; they are applied by setting entry 1 to 0 (“total cross section”), and specifying entries 2,21,22 and/or 3,31,32. Note that entry 4 changes between rapidity $y$ and pseudo-rapidity $\eta$.

Block SCALES defines the renormalization and factorization scales relative to the Higgs mass. In case of $p_T$ distributions the user can switch to a $p_T$ dependent renormalization and factorization scale. In accordance with Section 2.4 Block RENORMBOT offers different options for the renormalization of the bottom Yukawa coupling; three options are currently implemented: $m_b^Y \in \{m_b^{\text{OS}}, m_b^{\overline{\text{MS}}}(m_b), m_b^{\overline{\text{MS}}}()\}$. Entry 4 allows to fix the pole bottom mass $m_b^{\text{OS}}$ by hand. Block PDFSPEC contains the PDF sets in the notation of LHAPDF, consisting of the name of the PDF grid file, and the set number. Block VEGAS specifies integration parameters; note that distributions or cuts require higher numerical precision than the total cross section in order to reach comparable accuracy in the final result. Finally, Block FACTORS allows for additional factors in the Yukawa couplings of the fermions. We add that also charm-quark contributions can be taken into account by setting the corresponding factor to 1. Then the $c$-quark contributions at (N)LO are added using the on-shell value $m_c^{\text{OS}}$ calculated from $m_c(m_c)$ as done for the on-shell bottom-quark mass. In case of the MSSM, for which a detailed prescription follows, the charm-quark contributions can be added as well.

\textsuperscript{6}Apart from the scalar Higgs boson of the actual SM, SusHi also provides results for a pseudo-scalar Higgs-like particle whose coupling to fermions is obtained from the corresponding MSSM couplings by setting $\tan \beta = 1$. 

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6.4.2. 2-Higgs-Doublet-Model

The input of the 2HDM closely resembles the SM input. The user has to specify in addition the Higgs under consideration using Entry 2 of Block SUSHI. Moreover the corresponding Higgs mass has to be given in Block MASS together with the Higgs mixing angle $\alpha$ in Block ALPHA and the ratio of the vacuum expectation values $\tan \beta$ in Block MINPAR. The different types of the 2HDM as explained in Section 2.2 can be distinguished in Block 2HDM:

<table>
<thead>
<tr>
<th>Block SUSHI</th>
<th># model: 0 = SM, 1 = MSSM, 2 = 2HDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Block 2HDM</td>
<td># 2HDM version according to arxiv:1106.0034</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(1=I,2=II,3=III,4=IV)</td>
</tr>
<tr>
<td>Block MINPAR</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.0d0</td>
</tr>
<tr>
<td>Block ALPHA</td>
<td></td>
</tr>
<tr>
<td>-5.0e-01</td>
<td># mixing in Higgs sector</td>
</tr>
<tr>
<td>Block MASS</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>125.0000000d0</td>
</tr>
<tr>
<td>35</td>
<td>150.0000000d0</td>
</tr>
<tr>
<td>36</td>
<td>300.0000000d0</td>
</tr>
</tbody>
</table>

6.4.3. Minimal Supersymmetric Standard Model

In case of the MSSM, the input file contains a number of additional Blocks. We show them here, together with the relevant modifications of the SM version:

<table>
<thead>
<tr>
<th>Block SUSHI</th>
<th># model: 0 = SM, 1 = MSSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Block EXTPAR</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>800.0d0</td>
</tr>
<tr>
<td>11</td>
<td>2000.0d0</td>
</tr>
<tr>
<td>12</td>
<td>2000.0d0</td>
</tr>
<tr>
<td>23</td>
<td>200.0d0</td>
</tr>
<tr>
<td>26</td>
<td>130.0d0</td>
</tr>
<tr>
<td>43</td>
<td>1000.0d0</td>
</tr>
<tr>
<td>46</td>
<td>1000.0d0</td>
</tr>
<tr>
<td>49</td>
<td>1000.0d0</td>
</tr>
<tr>
<td>Block FEYNHIGGS</td>
<td>FeynHiggs specific input</td>
</tr>
<tr>
<td>1</td>
<td>0.0d0</td>
</tr>
<tr>
<td>2</td>
<td>200.0d0</td>
</tr>
<tr>
<td>3</td>
<td>2000.0d0</td>
</tr>
<tr>
<td>4</td>
<td>1000.0d0</td>
</tr>
<tr>
<td>Block RENORMBOT</td>
<td>Renormalization of the bottom sector</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Block FACTORS</td>
<td>factor for yukawa-couplings: c</td>
</tr>
<tr>
<td>1</td>
<td>0.0d0</td>
</tr>
<tr>
<td>2</td>
<td>1.0d0</td>
</tr>
<tr>
<td>3</td>
<td>1.0d0</td>
</tr>
<tr>
<td>4</td>
<td>1.0d0</td>
</tr>
<tr>
<td>5</td>
<td>1.0d0</td>
</tr>
</tbody>
</table>
Entry 2 of Block SUSHI now distinguishes between the three MSSM Higgs bosons. Entry 5 allows to add approximated NNLO stop contributions for the light Higgs h. Block MINPAR, entry 3 defines the value of \( \tan \beta \). Block EXTPAR fixes the parameters of the third family of quarks and squarks in the MSSM. If the Block FEYNHIGGS is present, SusHi has to be linked to FeynHiggs (see Section 6.3) which will then be used to calculate the Higgs masses from the parameters of that Block. In addition to the SM version, Block RENORMBOT provides various ways of resumming \( \ell_\beta \)-enhanced effects for the on-shell bottom Yukawa coupling, see Section 2.4. For a running coupling (entry \( 1 \in \{1, 2\} \)), resummation of those effects is always performed as shown in Eq. (15). Block RENORMSBOT provides the choice between the various options of Tab. 3. The alternative to Block FEYNHIGGS is the specification of the Higgs masses and the Higgs mixing angle \( \alpha \) by hand; for example:

| Block | ALPHA | -2.58961078E-01 | # mixing in Higgs sector |
| Block | MASS | 25 125.216431E+00 | # Higgs mass h |
| Block | 26 303.288802E+00 | # Higgs mass H |
| Block | 36 130.000000E+00 | # Pseudoscalar Higgs mass A |

In this case, it is the user’s responsibility to assure consistency of the Higgs mass and the other parameters. However, this option allows one to use \( H_3 m \) [80], for example, in order to take into account three-loop effects to the SUSY Higgs mass [79–81].

Example input files can be found in the subfolder example, namely a SM, a 2HDM and several MSSM input files, the latter for the usage with and without FeynHiggs. In case different soft-breaking masses for the slepton and squark generations should be specified we also refer to those input files and the webpage [1].

### 6.4.4. Output file

SusHi outputs the results of the calculation as well as some key parameters derived from the input in the same format as the input file. A typical example is shown here:

| Block | SUSHIggh | # Bon appetit |
| Block | 1 9.60254100E+00 | # ggh XS in pb |
| Block | SUSHIbbh | # Bon appetit |
| Block | 1 4.90606675E+00 | # bhh XS in pb |
| Block | XSGGH | # ggh MSSM-Cross sec. in pb (w/o EW) |
| Block | 2 7.16713162E+00 | # NLO |
| Block | XSGGHEFF | # ggh MSSM-Cross sec. |
| Block | 1 7.99562808E+00 | # ggh@NLO MSSM |
| Block | 2 1.01134207E+01 | # ggh@NNLO MSSM |
| Block | 3 4.43157467E-02 | # electroweak factor |
| Block | XSBBH | # bhh MSSM-Cross sec. in pb |
| Block | 1 7.07152627E+00 | # LO |
| Block | 2 6.24423880E+00 | # NLO |
| Block | 3 4.90606675E+00 | # NNLO |
| Block | HGGSUSY | # couplings of light Higgs h to 3. generation |
| Block | MASSOUT | 5 4.20000000E+00 | # m_b(m_b), MSbar |
| Block | 25 1.11485364E+02 | # MSSM-Mh in GeV |
| Block | ALPHA | -8.66408199E-01 | # alpha |


The main result for the gluon fusion cross section, containing all corrections specified by the user in the input file, is given as entry 1 in Block SUSHIggh, the one for bottom-quark annihilation in Block SUSHIbbh. Individual contributions to the cross sections are listed in Block XSGGH and Block XSBBH; their meaning should be obvious from the comment in the output file. Note that the results denoted “LO” etc. mean that the LO partonic cross section is convolved with the PDF set given in entry 1 of Block PDFSPEC in the input file.

For gluon fusion, the Block XSGCHEFF contains the NNLO top-(s)quark results as obtained by ggh@nnlo, and the electro-weak correction factor as determined in Section 3.4.

In addition, Block HGGSUSY lists the non-resummed MSSM couplings of the quarks (and squarks) to the Higgs boson under consideration. Block MASSOUT gives the relevant SM and SUSY masses as well as the Higgs mass. Not shown above is the Block INTERNALMASSES, which provides the different bottom masses entering the calculation of gluon-fusion cross sections, and ALPHASOUT showing the used values of $\alpha_s(\mu_R)$ taken from the PDF sets at (N)LO.

Since SusHi makes use of various results from different groups, the output file contains a run-specific list of references. We kindly ask the user to cite the mentioned references. Finally all output files have the reprocessed input file attached at the end. Output files can be used as input files as well.

7. Conclusion

In this article we described the Fortran code SusHi for the calculation of the cross section for Higgs production in gluon fusion and bottom-quark annihilation at hadron colliders. It works in the SM, the 2HDM and the MSSM, evaluates inclusive cross sections, distributions, and allows for kinematical cuts on the Higgs 4-momentum. It includes higher order QCD and electro-weak corrections and takes into account the effect from squarks and gluinos.

SusHi allows one to choose among various renormalization schemes for the sbottom sector and the bottom Yukawa coupling, and includes the resummation of $\tan \beta$-enhanced effects. For the calculation of the Higgs mass in the MSSM SusHi can be linked to FeynHiggs. SusHi can be downloaded from Ref. [1].

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We are grateful to the authors of Refs. [37, 42, 44, 45, 70], in particular Pietro Slavich for many helpful comments on our program, this manual, for many cross checks, and for
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fruitful discussions. Regarding the interpretation of FeynHiggs internal and output variables our thanks go to Thomas Hahn, Sven Heinemeyer, Heidi Rzehak and Georg Weiglein. Concerning the electro-weak contributions of light quarks, we thank Alessandro Vicini and Giuseppe Degrassi for comparing numbers.

Appendix A. Formulas: Higgs-squark couplings

In this section we present the couplings of the three neutral Higgs bosons $\phi$ of the MSSM to the quarks and squarks being implemented in SusHi. The relevant Feynman rules can be written in the form

$$\phi \quad \begin{cases} \quad \phi \quad \end{cases} = \frac{i m_q}{v} g_{\phi}^q \quad \text{and} \quad \phi \quad \begin{cases} \quad \phi \quad \end{cases} = \frac{i m_{\tilde{q}}}{v} g_{\phi,ij}^\tilde{q}, \quad (A.1)$$

where $v = 2m_W/g = 1/\sqrt{2G_F} = \sqrt{v_s^2 + v_c^2}$. The couplings $g_{\phi}^q$ of the Higgs boson $\phi$ to the quarks $q$ with respect to the SM Higgs boson coupling were already presented in Tab.2. The couplings $g_{\phi,ij}^\tilde{q}$ of the squarks to the light and heavy Higgs can be split in the form

$$g_{\phi,ij}^\tilde{q} = g_{\phi,EW}^\tilde{q} + g_{\phi,\mu}^\tilde{q} + g_{\phi,\alpha}^\tilde{q}. \quad (A.2)$$

In case of the light Higgs $h$ we obtain for the couplings:

$$g_{h,EW}^{11} = c_{\theta_1}^{EW} c_{\theta_2}^{EW} + c_{\theta_1}^{EW} c_{\theta_2}^{EW} s_{\theta_1}^{EW}$$

$$g_{h,EW}^{22} = c_{\theta_1}^{EW} s_{\theta_2}^{EW} + c_{\theta_1}^{EW} c_{\theta_2}^{EW}$$

$$g_{h,EW}^{12} = g_{h,EW}^{21} = \frac{1}{2} \left( c_{\theta_2}^{EW} - c_{\theta_1}^{EW} \right) s_{\theta_1}^{EW}$$

$$g_{h,\mu}^{11} = -g_{h,\mu}^{22} = \frac{\mu \cos(\alpha - \beta)}{m_t} s_{\theta_1}^{EW}$$

$$g_{h,\mu}^{12} = g_{h,\mu}^{21} = \frac{\mu \cos(\alpha - \beta)}{c_{\theta_1}^{EW}} s_{\theta_1}^{EW}$$

$$g_{h,11}^{11} = \frac{c_\alpha}{s_\beta} \left( 2 + \frac{m_{t_1}^2 - m_{t_2}^2}{2m_t^2} s_{2\theta_1}^2 \right)$$

$$g_{h,11}^{22} = \frac{c_\alpha}{s_\beta} \left( 2 - \frac{m_{t_1}^2 - m_{t_2}^2}{2m_t^2} s_{2\theta_1}^2 \right)$$

$$g_{h,12}^{12} = g_{h,21}^{11} = \frac{\alpha}{c_\beta} \left( 2 + \frac{m_{b_1}^2 - m_{b_2}^2}{2m_b^2} s_{2\theta_b}^2 \right)$$

$$g_{h,12}^{22} = g_{h,21}^{11} = \frac{\alpha}{c_\beta} \left( 2 - \frac{m_{b_1}^2 - m_{b_2}^2}{2m_b^2} s_{2\theta_b}^2 \right)$$

In case of the heavy Higgs $H$ we obtain for the couplings:

$$g_{H,EW}^{11} = c_{\theta_1}^{EW} c_{\theta_2}^{EW} + c_{\theta_1}^{EW} c_{\theta_2}^{EW} s_{\theta_1}^{EW}$$

$$g_{H,EW}^{22} = c_{\theta_1}^{EW} s_{\theta_2}^{EW} + c_{\theta_1}^{EW} c_{\theta_2}^{EW}$$

$$g_{H,EW}^{12} = g_{H,EW}^{21} = \frac{1}{2} \left( c_{\theta_2}^{EW} - c_{\theta_1}^{EW} \right) s_{\theta_1}^{EW}$$

$$g_{H,\mu}^{11} = -g_{H,\mu}^{22} = \frac{\mu \cos(\alpha - \beta)}{m_t} s_{\theta_1}^{EW}$$

$$g_{H,\mu}^{12} = g_{H,\mu}^{21} = \frac{\mu \cos(\alpha - \beta)}{c_{\theta_1}^{EW}} s_{\theta_1}^{EW}$$

$$g_{H,11}^{11} = \frac{c_\alpha}{s_\beta} \left( 2 + \frac{m_{t_1}^2 - m_{t_2}^2}{2m_t^2} s_{2\theta_1}^2 \right)$$

$$g_{H,11}^{22} = \frac{c_\alpha}{s_\beta} \left( 2 - \frac{m_{t_1}^2 - m_{t_2}^2}{2m_t^2} s_{2\theta_1}^2 \right)$$

$$g_{H,12}^{12} = g_{H,21}^{11} = \frac{\alpha}{c_\beta} \left( 2 + \frac{m_{b_1}^2 - m_{b_2}^2}{2m_b^2} s_{2\theta_b}^2 \right)$$

$$g_{H,12}^{22} = g_{H,21}^{11} = \frac{\alpha}{c_\beta} \left( 2 - \frac{m_{b_1}^2 - m_{b_2}^2}{2m_b^2} s_{2\theta_b}^2 \right)$$

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Therein we made use of the abbreviations $s_x = \sin x$ and $c_x = \cos x$ and defined:

\[
\begin{align*}
\tilde{c}_{E_t,1}^W &= -\frac{m_t^2}{m_t^2} \left(1 - \frac{4}{3} s_{\theta_W}^2\right) \sin(\alpha + \beta) \\
\tilde{c}_{E_t,2}^W &= -\frac{m_t^2}{m_t^2} \frac{2}{3} s_{\theta_W}^2 \sin(\alpha + \beta) \\
\tilde{c}_{E_b,1}^W &= \frac{m_b^2}{m_b^2} \left(1 - \frac{2}{3} s_{\theta_W}^2\right) \sin(\alpha + \beta) \\
\tilde{c}_{E_b,2}^W &= \frac{m_b^2}{m_b^2} \frac{2}{3} s_{\theta_W}^2 \sin(\alpha + \beta)
\end{align*}
\] (A.11)

The couplings to the heavy Higgs $H$ are easy to obtain by the replacement $\alpha \rightarrow \alpha - \frac{\pi}{2}$ in the previous formulas. In case of the CP-odd Higgs $A$ the couplings are given by:

\[
\begin{align*}
g_{A,t,11} &= g_{A,t,22} = g_{A,b,11} = g_{A,b,22} = 0 \\
g_{A,t,12} &= -g_{A,t,21} = \frac{1}{t_\beta} \left( \frac{m_t^2 - m_b^2}{2 m_t^2} s_{2\theta_t} + \mu \left(1 + \frac{1}{t_\beta^2}\right) \right) \\
g_{A,b,12} &= -g_{A,b,21} = t_\beta \left( \frac{m_b^2 - m_t^2}{2 m_b^2} s_{2\theta_b} + \mu \left(1 + \frac{1}{t_\beta^2}\right) \right)
\end{align*}
\] (A.13)

We add that $m_b$ is partially interpreted as the bottom mass in the sbottom sector, namely where it is meant to be part of the Higgs-sbottom coupling.

References

[1] SusHi can be downloaded from: http://sushi.hepforge.org/


[39] A. Pak, M. Steinhauser and N. Zerf, “Towards Higgs boson production in gluon fusion to NNLO in the
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[LHC Higgs Cross Section Working Group Wiki Page]


[56] R. Harlander, http://www.robert-harlander.de/software/ggh@nnlo


[63] R. Harlander, http://www.robert-harlander.de/software/bbh@nnlo


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M. Spira, “HIGLU: A program for the calculation of the total Higgs production cross-section at hadron colliders via gluon fusion including QCD corrections,” hep-ph/9510347.


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